Journal of Business & Economic Statistics, July 1989, Vol. 7, No. 3

Reverse Regression, Collinearity, and Employment Discrimination

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Apparently contradictory results between direct and reverse regression in employment-discrimination data analysis are a manifestation of collinearity in the data. An easily implemented guideline that alerts the analyst to the presence of contaminating collinearity is illustrated with employment data from Title VII litigation.

KEY WORDS: Multicollinearity; Multiple regression; Title VII methodology; Unfairness.

1. INTRODUCTION

Statisticians, economists, and members of the legal and business communities continue to argue the merits of reverse regression as a method for assessing equality of pay for minorities and women (among others, Conway and Roberts 1983, 1984; Dempster 1986; Goldberger 1984; Greene 1984). Reverse regression is the technique of regressing job qualifications on salary and sex. Hence the roles of qualifications and salary as the dependent and independent variables are reversed from the usual approach. The aim is to measure Type 2 unfairness, the difference in qualifications between protected and unprotected classes at the same salary level. Paradoxically, reverse regression tends to indicate the existence of unfairness against unprotected and protected classes simultaneously.

Conway and Roberts (1983) advocated reverse regression as "necessary to help decide whether males or females have been treated fairly" (p. 76). This published opinion has provoked considerable comment, including Greene's (1984) statement, "The technique of reverse regression not only fails to address the question, it introduces potentially misleading information into the debate" (p. 117).

This article illustrates that the conflict between direct and reverse regression can be resolved with recognition of another problem—collinearity in the data. When direct and reverse regression yield contradictory results for a given data set, a collinear structure must exist in the data between protected status and measures of job qualifications. When such collinearity is present, regression with the ordinary least squares (OLS) criterion, whether direct or reverse, introduces potentially misleading information anyway.

Moreover, collinearity is widespread in employment data. For example, in traditionally male-dominated professions, such as engineering and law, where most

Table 1. Regression Results for the University of Texas at El Paso Data Using Direct Regression; Analysis of Variance

Source	DF	Sum of squares	Mean square	F value	Prob > F
Model	10	2,935,555,727	293,555,573	122,219	.0001
Error	325	780,613,949	2,401,889.08		
C Total	335	3,716,169,677			

NOTE: Bound on collinearity is as follows: $\bar{y}_f - \bar{y}_m$ = 13,978.56–17,241.85, $c = -535.94$ (significance level .0263), $\alpha = 6.089$, $R^2_{\hat{y},x,d} = .7899$, bound = 0, and $R^2_{\hat{x},d} = .1956$. The dependent variable is salary; the root mean squared error is 1,549.803; the dependent mean is 16,620.27; the coefficient of variation is 9.324774; the R square is .7899; the adjusted R square is .7835.

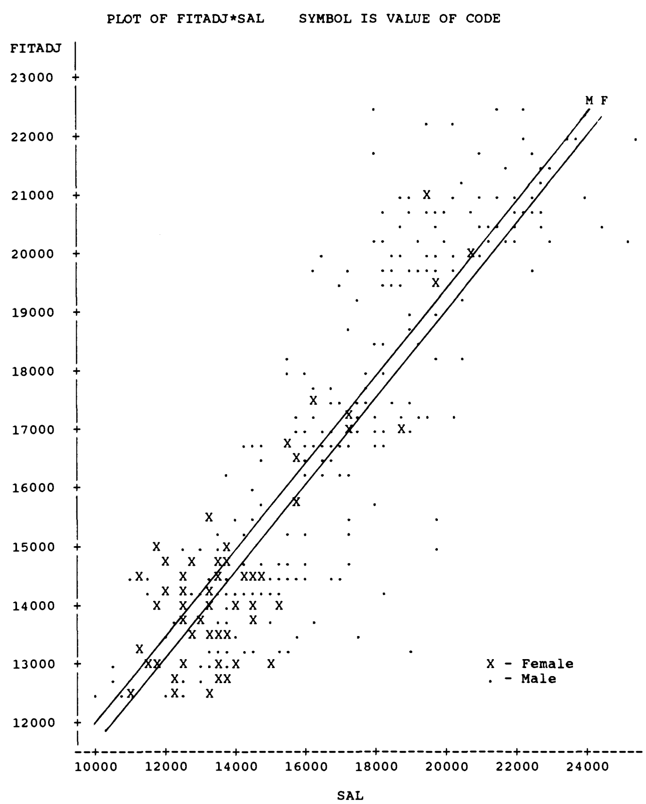

Figure 1. Scattergram of Case Data.

Table 2. Regression Results for the University of Texas at El Paso Data Using Direct Regression; Parameter Estimates

Variable	DF	Parameter estimate	Standard error	T for HO: Parameter = 0	Prob > |T|	Variable	DF	Tolerance	Variance inflation
Intercep	1	12,450.45440	289.53081	43.002	.0001	Intercep	1	.	0
AsoProf	1	2,323.10924	250.54045	9.272	.0001	AsoProf	1	.56891487	1.75773223
PhD	1	1,771.61749	292.26103	6.062	.0001	PhD	1	.41808173	2.39187682
Engr	1	1,770.98350	303.83455	5.829	.0001	Engr	1	.95230315	1.05008578
Bus	1	870.98691	341.11798	2.553	.0111	Bus	1	.96340931	1.03798042
YSHI	1	68.22050520	14.42641177	4.729	.0001	YSHI	1	.40197980	2.48768718
YEMP	1	−41.82574944	21.04432229	−1.988	.0477	YEMP	1	.33192160	3.01275964
Sex	1	−535.94385	240.06443	−2.233	.0263	Sex	1	.80442899	1.24311781
YIRank	1	48.68600972	27.58112863	1.765	.0785	YIRank	1	.32862184	3.04301138
Acct	1	1,309.14164	567.86735	2.305	.0218	Acct	1	.95375010	1.04849268
I1	1	5,105.60487	300.51077	16.990	.0001	I1	1	.39544211	2.52881516

NOTE: See note to Table 1.

females have entered the field in the last 10 years or so, length of employment is clearly distributed differently for men and women. Condition indexes (Belsley, Kuh, and Welsch 1980, pp. 98–115), the most accessible and frequently used measures of collinearity, are not sufficiently sensitive to detect levels of collinearity that introduce misleading results in employment-data analysis. We present a guideline for detecting contaminating collinearity that is easily obtained from any multiple-regression package and illustrate its use with employment data from Title VII litigation. This simple rule of thumb alerts the analyst to the collinear structure of the data and the need to assess fairness of pay with methods other than OLS regression.

2. REGRESSION MODELS AND A BOUND ON COLLINEARITY

Let Y be a reward or incentive measure, $X = (1, X_1, \ldots, X_k)'$ be a vector of job qualification or performance measures, d be a binary indicator of the protected class of interest (e.g., $d = 1$ indicates female or minority), and ε be a random variable with zero mean representing error. Thus a classical regression model of the form

$$Y = X\beta + d\gamma + \varepsilon, \tag{1}$$

with $\beta = (\beta_0, \ldots, \beta_k)$, is traditionally used to ascertain Type 1 unfairness, which is the difference in expected salaries between protected and unprotected classes at the same level of job qualification and performance measure. In the absence of Type 1 unfairness, $\gamma = 0$. The analysis begins with a data estimate of (1):

$$\hat{Y} = Xb + dc, \tag{2}$$

where b and c are OLS estimates of β and γ. Note, however, that for Title VII analysis the possibility of an inherent relationship between X and d could invalidate (1) as an appropriate model of reward or incentive.

The reverse-regression model advocated by Conway and Roberts is of the form

$$Xb = Y\beta^* + d\gamma^* + \varepsilon. \tag{3}$$

The least squares estimate of (3) yields

$$\hat{Xb} = Yb^* + dc^*, \tag{4}$$

where b^* and c^* are OLS estimates of β^* and γ^*.

In the case in which X and d are orthogonal,

$$c = -c^* R^2_{y,x \cdot d}. \tag{5}$$

Thus, in the complete absence of collinearity, c and c^* must be of opposite sign, which means that Type 1 and Type 2 unfairness cannot exist simultaneously.

Suppose that c is less than 0, apparently indicating Type 1 unfairness. Now assume $c^* < 0$ also, indicating Type 2 unfairness and contradictory results; that is,

$$(\bar{y}_f - \bar{y})(1 - R^2_{y,x \cdot d})/(1 - P) - c < 0, \tag{6}$$

where $\bar{y}$ is the sample mean salary for all observations, $\bar{y}_f$ is the sample mean salary for members of the pro-

Table 3. Regression Results for the University of Texas at El Paso Data Using Direct Regression; Collinearity Diagnostics

Number	Eigenvalue	Condition number	Var prop Intercep	Var prop AsoProf	Var prop PhD	Var prop Engr	Var prop Bus	Var prop YSHI	Var prop YEMP	Var prop Sex	Var prop YIRank	Var prop Acct	Var prop I1
1	5.004059	1.000000	.0030	.0047	.0032	.0046	.0031	.0048	.0040	.0037	.0048	.0008	.0046
2	1.099427	2.133428	.0009	.0082	.0011	.1383	.0023	.0000	.0000	.2373	.0007	.2118	.0317
3	1.090169	2.142468	.0002	.1386	.0015	.2027	.1108	.0031	.0005	.0109	.0019	.0361	.0333
4	.968745	2.272776	.0001	.0465	.0000	.0008	.2241	.0006	.0000	.0170	.0000	.4337	.0488
5	.924896	2.326027	.0000	.0139	.0025	.0071	.4580	.0039	.0022	.1503	.0006	.2095	.0012
6	.730464	2.617350	.0000	.0716	.0030	.6214	.1491	.0004	.0022	.1446	.0028	.0235	.0001
7	.647924	2.779069	.0059	.0025	.0540	.0144	.0055	.0193	.0201	.0682	.0515	.0324	.0633
8	.232725	4.637030	.0193	.3424	.0896	.0004	.0158	.1387	.0017	.0188	.1110	.0026	.2275
9	.166453	5.482970	.1427	.1550	.0001	.0022	.0019	.2705	.0417	.2518	.1192	.0010	.1988
10	.094119	7.291590	.0567	.0271	.0378	.0001	.0015	.1512	.6962	.0828	.4179	.0001	.0014
11	.041020	11.044974	.7712	.1895	.8073	.0081	.0278	.4075	.2313	.0146	.2898	.0484	.3892

NOTE: See note to Table 1.

Table 4. Regression Results for the University of Texas at El Paso Data Using Reverse Regression; Analysis of Variance

Source	DF	Sum of squares	Mean square	F Value	Prob > F
Model	2	2,181,163,668	1,090,581,834	617.573	.0001
Error	333	588,050,070	1,765,916.13		
C Total	335	2,769,213,737			

NOTE: See note to Table 1. $c^* = -269.05$ (significance level, .1796). The dependent variable is FITADJ = Xb; the root mean squared error is 1,328.878; the dependent mean is 16,722.36; the coefficient of variation is 7.946713; the R square is .7876; and the adjusted R square is .7864.

tected class, P is the proportion of the sample observations that belong to the protected class, and $R^2_{y,x\cdot d}$ is the coefficient of determination in the regression of Y on X after "netting" out the effect of d. From relationships among partial and multiple correlation coefficients (Theil 1971), it follows that (6) yields

$$R^2_{x,d} > \frac{c - (\bar{y}_f - \bar{y}_m)(1 - R^2_{y,x,d})}{c}, \tag{7}$$

where y_m is the sample mean for members of the unprotected class. Let

$$\alpha = (\bar{y}_f - \bar{y}_m)/c. \tag{8}$$

The upper bound on collinearity is

$$1 - \alpha(1 - R^2_{y,x,d}), \tag{9}$$

provided that this bound is positive, and 0 otherwise. If $R^2_{x,d}$ does not exceed this bound on collinearity, contradictory conclusions with respect to Type 1 and Type 2 fairness cannot result.

The parameters in this collinearity bound are all readily available from any software package that has a multiple-regression subroutine. $R^2_{x,d}$ is obtained as 1 less the tolerance of d.

3. AN EMPIRICAL EXAMPLE

Consider a regression model from an equal-pay suit brought by women faculty members against the University of Texas at El Paso. Data are for the 1975–1976 academic year. The quantitative variables in this model are years since highest degree, years employed, and years in rank. Indicator variables included are Associate Professor (AsoProf), Ph.D. Degree, Engineering Faculty, Business Faculty, Accounting Faculty, Interaction of Full Professor and Ph.D., and Sex. Data for more traditional measures of academic performance, such as pages published in refereed journals, were not stipulated to by the court at the time of trial. Sample size is 336. Regression coefficients and other statistics appear in Tables 1–6.

The direct regression coefficient for Sex, c, is $-\$536$, with an observed significance level of .0263, apparently suggesting Type 1 unfairness. The largest condition index is 11.045, ordinarily giving no indication of damaging collinearity. The coefficient of determination for sex and X, the vector of predictor variables, is .20. The mean salary for women is \$13,978.56; the mean for men is \$17,241.85. Taking the ratio of this difference to c, an α value of 6.089 is obtained. The model coefficient of determination is .79, yielding 0 as a bound on collinearity. The association between X and d exceeds the bound on collinearity and signals that the regression model merits reexamination. At this point, one should pursue alternative methods of analysis.

The reverse-regression results in this case are indeed contradictory. The estimate of c^* is $-\$269.05$, indicating simultaneous unfairness of Types 1 and 2. (The significance level of c^* is relatively high, .1796.) Consider Figure 1, a scatterplot of Xb, the dependent variable in the reverse regression, with Salary by Sex. The sample reverse regression models for men and women are superimposed on the data. In the area bounded by \$16,000 on both axes, the distribution of males and females in the data appears essentially symmetric, giving no evidence of Type 2 unfairness. For salary levels higher than \$16,000, only seven women appear in the data set. Four of these seven employees show observed productivity (Xb) higher than the expected value ($\hat{X}b$) for men at the given salary level, negating any evidence of Type 2 unfairness. The suggested Type 2 unfairness in fact reflects the uneven distribution of women in the data set over the entire range of salaries.

Using the same scatterplot, Type 1 unfairness indicated in the direct regression seems to exist only at estimated productivity levels less than \$18,000. The observed difference in means is exacerbated by values that

Table 5. Regression Results for the University of Texas at El Paso Data Using Reverse Regression; Parameter Estimates

Variable	DF	Parameter estimate	Standard error	T for HO: parameter = 0	Prob > |T|	Tolerance	Variance inflation
Intercep	1	4253.26514	415.19870	10.244	.0001	.	0
Sal	1	.75331740	.02362306	31.889	.0001	.85153451	1.17435053
Sex	1	−269.05313	200.06868	−1.345	.1796	.85153451	1.17435053

NOTE: See note to Table 4.

Table 6. Regression Results for the University of Texas at El Paso Data Using Reverse Regression; Collinearity Diagnostics

Number	Eigenvalue	Condition number	Var prop Intercep	Var prop Sal	Var prop Sex
1	2.237151	1.000000	.0058	.0059	.0530
2	.746968	1.730600	.0025	.0051	.7602
3	.015881	11.869031	.9918	.9890	.1868

NOTE: See note to Table 4.

positively skew the distribution of men's salaries, a cluster of about five men who are earning relatively high salaries for untypically low levels of estimated productivity. Thus the salary differential indicated by the direct regression does not apply across the range of salaries.

4. CONCLUSION

This article has presented an easily implemented guideline for ascertaining unacceptable levels of collinearity for regression analysis with employment data. The guideline is applied to a real data set from Title VII litigation against the University of Texas at El Paso. With this data set, the suggested bound on collinearity is exceeded and direct and reverse regressions give contradictory results. Examination of the scatterplot from the reverse regression model, however, indicates that there is in fact no Type 2 unfairness. Type 1 unfairness appears only in certain ranges of the data.

ACKNOWLEDGMENTS

We would like to acknowledge the cooperation of the Office of the General Counsel of the University of Texas System. In particular, we would like to thank W. O. Schultz III, Robert Giddings, and Lonnie Zwiener.

[Received June 1987. Revised November 1988.]

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